Random walks with local memory

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The theme of this talk is walks in a random environment of "signposts" altered by the walker. I'll focus on three related examples:

1. Rotor walk on Z^2. Your initial signposts are independent with the uniform distribution on {North,East,South,West}. At each step you rotate the signpost at your current location clockwise 90 degrees and then follow it to a nearest neighbor. Priezzhev et al. conjectured that in n such steps you will visit order n^{2/3} distinct sites. I'll outline an elementary proof of a lower bound of this order. The upper bound, which is still open, is related to a famous question about the path of a light ray in a grid of randomly oriented mirrors. This part is joint work with Laura Florescu and Yuval Peres.

2. p-rotor walk on Z. In this walk you flip the signpost at your current location with probability 1-p, and then follow it. I'll explain why your scaling limit will be a Brownian motion perturbed at its extrema. This part is joint work with Wilfried Huss and Ecaterina Sava-Huss.

3. p-rotor walk on Z^2. Rotate the signpost at your current location clockwise with probability p and counterclockwise with probability 1-p, and then follow it. This walk "organizes" its environment by destroying cycles of signposts. A native environment – stationary in time, from your perspective as the walker – is an orientation of the uniform spanning forest, plus one additional edge. This part is joint work with Swee Hong Chan, Lila Greco, and Peter Li: https://arxiv.org/abs/1809.04710